A new approach to the measurement of turbulent fluxes in the lower atmosphere

By R. J. TAYLOR
C.S.I.R.O. Division of Meteorological Physics, Aspendale, Victoria, Australia

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A method is described for the measurement of turbulent shearing stress and vertical heat flux by way of 'structure functions'—the mean square velocity and temperature differences between two points a known distance apart. The method is particularly suitable for shipboard use because it does not require the measurement of velocity components relative to a fixed frame of reference. An analysis of the available observations, although they are not ideally suited to the purpose, is encouraging and points the way to further development of the method.

1. Introduction

It has several times been noted (e.g. by McCready 1953; Taylor 1955) that, when autocorrelations in horizontal wind velocity components at two points are measured in the lower atmosphere, they show the particular dependence on the distance between the two points which is characteristic of the inertial subrange of eddy sizes, up to unexpectedly large distances—often as much as several times the height of observation. A rather convincing explanation of this phenomenon has been put forward by Gifford (1959), but, whatever the cause, there is no doubt that this simple property does, in fact, hold good over an extensive range of distances.

Within this range, it is possible to derive expressions relating the vertical turbulent fluxes of heat and momentum with the mean square differences in temperature and velocity between two points—the so-called 'structure functions'. As pointed out by Deacon (1959), these could form the basis of a method for measuring the fluxes which would have obvious applications at sea since the only functions of velocity required would be differences. These, though affected to some extent by the ship's motion, could be measured more accurately than velocity components relative to a fixed frame of reference.

The theoretical basis of the method is set out below, and the available observations are used to test it. Although the observations are not entirely suited to the purpose, a preliminary assessment is possible.

2. Theoretical basis of the method

If we write
$$D_u(r) = \overline{[u(x)-u(x+r)]^2}, \tag{1}$$
where u is the velocity component in the downwind (x) direction and r is an increment of distance in the same direction, then, in the inertial subrange,
$$D_u(r) \propto \epsilon^{\frac{2}{3}} r^{\frac{2}{3}}, \tag{2}$$

where ϵ is the rate of dissipation of kinetic energy per unit of mass. Obukhov & Yaglom (1951) derived a value for the constant in (2) and wrote

$$D_u(r) = 1{\cdot}6\epsilon^{\frac{2}{3}}r^{\frac{2}{3}}. \tag{3}$$

Their derivation rests on a value of $-0{\cdot}4$ for the skewness in distribution of $[u(x)-u(x+r)]$. Some measurements by Stewart (1951) suggest that a rather smaller absolute value may be more appropriate and this would involve a larger constant in (3). For the present, equation (3) will be accepted as it stands, in the knowledge that some adjustment of this constant may later prove to be necessary.

Equation (3) links ϵ with a measurable function of wind velocities. As is well-known, ϵ can also be expressed by the equation

$$\epsilon = \frac{\tau}{\rho}\frac{\partial \overline{u}}{\partial z} + \frac{gH}{c_p \rho T} \tag{4}$$

(neglecting divergence of vertical diffusion of turbulent kinetic energy) where τ is the shearing stress, H is the vertical turbulent heat flux and the other symbols are standard notation. It is thus clearly possible, by way of equations (3) and (4) to estimate τ from measurements of D_u and the other quantities involved.

The distribution of temperature fluctuations in a turbulent flow has not received as much attention as the dynamically more interesting problem of the velocity fluctuations. The earliest relevant work appears to be that of Obukhov (1949) who discussed temperature structure functions of the form

$$D_T(\mathbf{r}) = \overline{[T(\mathbf{x}) - T(\mathbf{x}+\mathbf{r})]^2} \tag{5}$$

where $T(\mathbf{x})$ and $T(\mathbf{x}+\mathbf{r})$ are the temperatures at points having co-ordinates x_i and $x_i + r_i$ respectively ($i = 1, 2, 3$).

Obukhov introduces a dissipation function for temperature fluctuations

$$\chi = \kappa(\partial T/\partial x_i)^2, \tag{6}$$

where κ is the thermal diffusivity of the fluid. (The summation convention for subscripts applies in equation (6).) He assumes that there exists a range of eddy sizes for which D_T is a function of r, χ and ϵ only, and dimensional analysis then leads to the result that, for this range of sizes,

$$D_T(r) \propto \chi\epsilon^{-\frac{1}{3}}r^{\frac{2}{3}}. \tag{7}$$

Similarly, Corrsin (1951) obtained a spectrum of temperature fluctuation proportional to the minus five-thirds power of the wave-number, a result which is exactly equivalent to Obukhov's. In sharp contrast, however, is a paper by Inoue (1952) who, using a very different theoretical basis, reached the conclusion that $D_T(r)$ is proportional to $r^{\frac{4}{3}}$.

A relationship between χ and H was found by Tatarskii (1956) by assuming a logarithmic form of temperature profile and taking a simplified form of the equation of conservation of internal energy. It is possible, however, to show that his relationship is valid even when the profile is not logarithmic and when certain terms, neglected by him, are taken into account.

The equation of motion of a fluid of variable density (see Jeffreys 1952) is

$$\rho\frac{\partial u_i}{\partial t}+\rho u_j\frac{\partial u_i}{\partial x_j}+\frac{\partial p}{\partial x_i}+\rho\frac{\partial \phi}{\partial x_i}-\frac{\partial \sigma_{ij}}{\partial x_j}=0, \tag{8}$$

in which the summation convention applies. Here the u_i are the velocity components, ϕ is the geopotential and σ_{ij} is a viscous stress given in full by

$$\sigma_{ij}=\rho\nu\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}-\frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\right),$$

where ν is the kinematic viscosity and $\delta_{ij}=1$ or 0 according as $i=j$ or not. By multiplying equation (8) throughout by u_i, we obtain the equation for conservation of kinetic and potential energies:

$$\rho\frac{\partial}{\partial t}(\tfrac{1}{2}u_i^2)+\rho u_j\frac{\partial}{\partial x_j}(\tfrac{1}{2}u_i^2)+u_i\frac{\partial p}{\partial x_i}+\rho u_i\frac{\partial \phi}{\partial x_i}-u_i\frac{\partial \sigma_{ij}}{\partial x_j}=0. \tag{9}$$

If radiative transfer is neglected, the equation for the conservation of energy (see de Groot, 1951) is

$$\frac{\partial}{\partial t}\rho[c_v T+\tfrac{1}{2}u_i^2+\phi]+\frac{\partial}{\partial x_j}\rho u_j[c_v T+\tfrac{1}{2}u_i^2+\phi]-\lambda\frac{\partial^2 T}{\partial x_j^2}+\frac{\partial p u_j}{\partial x_j}-\frac{\partial \sigma_{ij}u_j}{\partial x_i}=0, \tag{10}$$

where λ is the thermal conductivity.

The equation for the conservation of internal energy is now derived from equations (9) and (10) by subtraction, using the equation of continuity, and is

$$\rho\frac{\partial}{\partial t}c_v T+\rho u_j\frac{\partial}{\partial x_j}c_v T+p\frac{\partial u_j}{\partial x_j}=\sigma_{ij}\frac{\partial u_i}{\partial x_j}+\lambda\frac{\partial^2 T}{\partial x_j^2}, \tag{11}$$

which reduces to

$$c_p\rho\frac{T}{\theta}\left(\frac{\partial \theta}{\partial t}+u_j\frac{\partial \theta}{\partial x_j}\right)=\sigma_{ij}\frac{\partial u_i}{\partial x_j}+\lambda\frac{\partial^2 T}{\partial x_j^2}, \tag{12}$$

where θ is the potential temperature.

If this equation is now multiplied through by a temperature fluctuation T' and averaged and, further, the assumptions of steadiness in time and horizontal uniformity are made, it reduces to

$$H\frac{\partial \bar{\theta}}{\partial z}+\tfrac{1}{2}c_p\frac{\partial}{\partial x_j}\overline{\{(\rho u_j)'\,T'^2\}}=\overline{\rho\epsilon T'}+\lambda\frac{\partial^2}{\partial x_j^2}(\tfrac{1}{2}\overline{T'^2})-\rho c_p\chi, \tag{13}$$

since fluctuations in T and θ are nearly equal. The order of magnitude of terms in this equation will be estimated later.

3. The observations

The only available data suitable for testing the proposed method are those of Swinbank (1955). He gives, *inter alia*, values of momentum flux (which is equivalent to the shearing stress) and heat flux, determined from the covariance of the vertical velocity component with the horizontal-downwind component and with the temperature respectively. Records of velocity and temperature fluctuations at one point as a function of time were made as part of this investigation and it is possible to estimate structure functions from them as follows.

Following G. I. Taylor (1938) it will be assumed that the autocorrelations behave as if the whole turbulent velocity and temperature fields were being swept along unchanged at the mean wind speed. The validity of this assumption in the lower atmosphere seems well established (see Gifford 1956; Taylor 1957). Thus structure functions for a distance r, where r is aligned along the mean wind direction, are to be taken as equal to mean square differences for a time interval σ where $r = \bar{u}\sigma$. This artifice is introduced solely in order to make use of existing data and is valid only if the point at which observations are made is fixed or in uniform motion. Aboard a ship, where the motion of the point of measurement would itself have autocorrelation, differences between two points would have to be measured directly.

Those records made by Swinbank at heights of 23 m and 29 m for which all the necessary subsidiary data were available have been used in this analysis. Observations at these heights only were used because previous work had suggested that at lower levels much of the interesting part of the eddy structure was of too short a period to be properly taken into account by the recording galvanometers used (natural period nominally 2·3 sec).

The stresses tabulated by Swinbank are available to test the present method. Unfortunately, the decision to use autocorrelations at 23 m and 29 m only involved the use of stresses measured at the same height for comparison purposes, and it has been shown by Deacon (1955) that the 5 min recording period used was not long enough for all the eddies contributing to the momentum flux at those heights to be fully accommodated. His comparison of the stress measurements there with those at 1·5 to 2 m led him to conclude that both sets of measurements are low by similar amounts, the former through the exclusion of low-frequency contributions, the latter through high-frequency cut-off. The latter effect has since been shown to amount to 20–30 % so, in what follows, Swinbank's stresses have been corrected by an increase of 30 % and are used as reference values denoted by τ.

No such correction to allow for apparent loss of flux due to the shortness of the observation period has been made to Swinbank's values of heat flux because the effect does not seem to be as severe here (Deacon 1955). The tabulated heat fluxes have been taken without change and are denoted by H.

It is clear from the above that these observations are far from ideal for testing the proposed structure-function method of estimating fluxes. Nevertheless, their application to this purpose is an obvious first step in verifying the method and will help in deciding whether a more refined programme of observations is justified.

4. First estimate of shearing stress

A total of 31 records with all necessary subsidiary observations was available and autocorrelations in u were calculated from them for time lags, σ, of about 1, 2, 5 and 9 sec for which experience had shown that equation (2) was fairly well obeyed at these heights. As a further test of this equation, the values of D_u so obtained were plotted logarithmically against σ so as to determine the index p in the proportionality

$$D_u \propto \sigma^p.$$

These values of p are shown in table 1 and the mean $0{\cdot}63 \pm 0{\cdot}03$ is in satisfactory agreement with the expected value.

Seven values of stress had previously been calculated (Taylor 1955) by the methods of §2 and had shown reasonable agreement on average with those measured by Swinbank. In this calculation, however, the second term on the

Date*	Run no.*	p	ϵ erg (g^{-1} sec^{-1})	$\partial\bar{u}/\partial z$ (sec^{-1})	T_1 (dyne cm^{-2})	τ (dyne cm^{-2})	T_1/τ
8. xi. 51	2	0·75	76·4	0·0291	3·00	1·83	1·64
9. i. 52	3	0·49	27·8	0·0134	−3·28	2·66	−1·23
16. i. 52	6	0·45	60·3	0·0386	1·20	3·22	0·37
21. i. 52	4	0·49	88·4	0·0156	3·22	2·35	1·37
4. ii. 52	2	0·75	39·6	0·0122	3·17	2·58	1·22
4. ii. 52	4	0·75	40·0	0·00513	−2·29	1·90	−1·21
4. ii. 52	6	0·75	132	0·00566	18·0	4·29	4·20
13. ii. 52	9	0·87	4·92	0·0751	0·11	0·56	0·20
13. ii. 52	11	0·81	2·35	0·0496	0·06	0·26	0·23
25. ii. 52	6	0·40	34·9	0·0532	0·09	2·06	0·05
25. ii. 52	11	0·73	45·5	0·0502	1·00	1·53	0·66
25. ii. 52	13	0·81	17·7	0·0390	0·34	2·26	0·15
25. ii. 52	15	0·51	53·1	0·0383	1·61	1·38	1·17
25. ii. 52	17	0·47	26·4	0·0485	0·76	1·34	0·57
25. ii. 52	19	0·60	12·6	0·0540	0·33	0·66	0·50
26. ii. 52	37	0·40	20·0	0·0239	0·73	0·44	1·66
26. ii. 52	39	0·73	21·4	0·0117	−0·35	2·72	−0·13
26. ii. 52	46	0·60	38·9	0·0239	1·16	1·90	0·62
26. ii. 52	48	0·78	79·7	0·0274	2·40	3·39	0·71
26. ii. 52	50	0·53	68·5	0·0241	2·16	2·46	0·88
3. iii. 52	4	0·65	101	0·0304	1·47	1·23	1·19
3. iii. 52	6	0·62	57·5	0·0153	−3·53	8·01	−0·44
3. iii. 52	10	0·47	53·5	0·00996	3·20	5·43	0·59
3. iii. 52	16	0·65	11·7	0·0414	0·43	0·35	1·23
4. iii. 52	48	0·55	33·1	0·0209	1·10	0·57	1·93
19. iii. 52	8	0·84	79·3	0·0575	2·03	4·70	0·43
19. iii. 52	10	0·70	80·9	0·0268	3·55	2·75	1·29
19. iii. 52	12	0·90	90·3	0·0328	2·90	4·71	0·62
19. iii. 52	16	0·67	37·2	0·0410	0·95	2·00	0·48
9. vii. 52	1	0·38	6·08	0·120	0·08	0·32	0·25
9. vii. 52	3	0·47	4·03	0·104	0·06	0·21	0·29
	Mean	0·63	—	—	—	—	0·69
Standard error of mean		0·03	—	—	—	—	0·18

* As shown by Swinbank (1955).

TABLE 1. Results of velocity autocorrelation analysis.

right of equation (4) was ignored since the observations concerned were mainly in the near-neutral condition. This comparison has now been extended by calculating stresses from the 31 new sets of structure functions.

Values of ϵ (as shown in table 1) were calculated according to equation (3) by plotting $D_u(\sigma)$ against $\sigma^{\frac{2}{3}}$ and fitting a straight line through the origin by eye. To calculate the stress then by equation (4) it is also necessary to know the heat flux and the velocity gradient at the height of observation. The values H tabulated by Swinbank were accepted for the former and the latter was calculated by fitting to the wind-speed observations at all available heights (which lay within the range 0·5–32 m) an interpolation formula of the form $\partial u/\partial z = az^{-\beta}$ where a and β are disposable constants. The values of shearing stress so calculated are denoted by T_1 and are shown in table 1.

The ratio T_1/τ was also calculated for each run (see table 1) and its mean is $0{\cdot}69 \pm 0{\cdot}18$. The expected value of unity is barely within the range of twice the standard deviation from that observed and it might appear that some adjustment of the constant in equation (3) is needed. This possibility is hardly surprising when it is recalled that the constant rests ultimately on a laboratory measurement of the skewness in distribution of $[u(x) - u(x+r)]$ which is itself open to doubt and that a very great change in scale is involved in its application here.

5. The heat flux

Little experimental evidence has been brought to bear on the question of whether $D_T(r)$ is proportional to $r^{\frac{2}{3}}$ or $r^{\frac{4}{3}}$. Inoue (1952) quotes some observations by Crain & Gerhardt (1951) and claims that they support his own proposed $r^{\frac{4}{3}}$ relationship. On the other hand, Shiotani (1955) and Tatarskii (1956), on the basis of observations which they adduce, both support the proposals of Obukhov and Corrsin. In view of this disagreement, it seemed desirable to examine the available observations on the point.

Autocorrelations at time lags, σ, were therefore calculated from temperature records of Swinbank's (1953) investigation. There were available 28 records suitable for this purpose which were also accompanied by the subsidiary data needed for heat flux estimation (see below). It was assumed that the limits of inertial subrange behaviour would be similar to those for velocity fluctuations and approximately the same values of σ were used. Logarithmic plotting of D_T against σ then gave the index p as shown in table 2. The mean p of $0{\cdot}64 \pm 0{\cdot}03$ unequivocally supports the 'two-thirds' law for the structure function, as expressed in equation (7). The constant of proportionality in this equation is, of course, not yet determined and a purely empirical value will later be derived.

To obtain some idea of the relative magnitudes of the terms in equation (13), four of the available records were analysed in detail. They were made in lapse conditions with an average Richardson number (at 1·5 m) of −0·051. On the average over these four runs, it was found that:

$$H\frac{\partial\bar{\theta}}{\partial z} = -14{\cdot}4\,\text{erg}\,{}^\circ\text{C}\,\text{cm}^{-3}\,\text{sec}^{-1},$$

$$\tfrac{1}{2}c_p\rho\overline{w'T'^2} = +0{\cdot}30\,\text{erg}\,{}^\circ\text{C}\,\text{cm}^{-2}\,\text{sec}^{-1},$$

$$\overline{\rho\epsilon(T'^2)^{\frac{1}{2}}} = +0{\cdot}02\,\text{erg}\,{}^\circ\text{C}\,\text{cm}^{-3}\,\text{sec}^{-1},$$

$$\lambda\frac{\partial^2}{\partial z^2}(\tfrac{1}{2}\overline{T'^2}) \approx 10^{-5}\,\text{erg}\,{}^\circ\text{C}\,\text{cm}^{-3}\,\text{sec}^{-1};$$

(w is the vertical velocity component).

As far as this evidence takes us, therefore, we are clearly justified in writing approximately

$$\chi = -\frac{H}{c_p \rho} \frac{\partial \overline{\theta}}{\partial z}. \tag{14}$$

This is equivalent to the equation finally arrived at by Tatarskii (1956).

Date	Run no.	p	χ $10^{-4}(°C)^2$ sec^{-1}	$\partial\theta/\partial z$ 10^{-6} °C cm^{-1}	H_1 ($mW\,cm^{-2}$)	H ($mW\,cm^{-2}$)	H_1/H
16. i. 52	6	0·65	7·36	−159	5·58	8·0	0·70
21. i. 52	2	0·51	25·6	−200	15·3	21·0	0·73
21. i. 52	4	0·45	25·2	−156	19·4	17·1	1·13
4. ii. 52	2	0·34	5·31	−136	4·70	2·7	1·74
4. ii. 52	4	0·58	11·1	−142	9·40	18·3	0·51
4. ii. 52	6	0·78	41·0	−192	25·6	17·4	1·47
13. ii. 52	9	0·65	0·233	98·5	−0·28	− 0·8	0·35
25. ii. 52	6	0·93	3·44	−125	3·31	11·3	0·29
25. ii. 52	11	0·47	0·415	−61·8	0·80	1·3	0·62
25. ii. 52	13	0·49	0·710	−17·4	4·90	2·4	2·04
25. ii. 52	15	0·73	0·221	−11·4	2·33	0·6	3·89
25. ii. 52	17	0·47	0·971	44·1	−2·64	−1·6	1·65
25. ii. 52	19	0·62	1·27	44·7	−3·41	−0·8	4·26
26. ii. 52	37	0·78	1·23	−138	1·07	2·0	0·54
26. ii. 52	39	0·97	1·89	−130	1·75	9·1	0·19
26. ii. 52	46	0·58	5·35	−145	4·42	5·8	0·76
26. ii. 52	48	0·75	13·1	−145	10·8	9·1	1·19
26. ii. 52	50	0·58	10·6	−150	8·49	9·2	0·92
3. iii. 52	4	0·78	24·0	−169	17·0	23·4	0·73
3. iii. 52	6	0·78	24·3	−217	13·4	37·7	0·36
3. iii. 52	10	0·78	1·89	−206	1·10	9·9	0·11
3. iii. 52	16	0·87	0·916	116	−0·95	−1·1	0·86
4. iii. 52	48	0·67	1·32	−42·9	3·69	5·1	0·72
19. iii. 52	10	0·58	0·864	−296	0·35	0·6	0·58
19. iii. 52	12	0·36	1·66	−114	1·75	4·0	0·44
19. iii. 52	16	0·90	1·26	−111	1·36	1·7	0·80
9. vii. 52	1	0·27	1·87	440	−0·51	−0·7	0·73
9. vii. 52	3	0·47	2·29	300	−0·92	−0·3	3·06
	Mean	0·64	—	—	—	—	1·12
Standard error of mean		0·03	—	—	—	—	0·20

TABLE 2. Results of temperature autocorrelation analysis.

Values of χ as shown in table 2 were calculated from the temperature autocorrelations by means of equation (7), tentatively taking the constant of proportionality as unity and using ϵ as given in table 1. Heat fluxes, H_1, were then obtained from equation (14). The potential temperature gradients required were derived from the original observations of temperature at heights within the range 0·5–30 m by fitting an interpolation formula of the form $\partial\theta/\partial z = az^{-\beta}$.

These values of H_1 are shown in table 2 and are compared with Swinbank's values, H, by calculating H_1/H. The mean value of H_1/H was $1{\cdot}12 \pm 0{\cdot}20$ and it thus appears that 1·12 is the best available estimate for the constant in equation (7). New heat fluxes, H_2, were calculated on this basis and it is obvious that the mean H_2/H must be $1{\cdot}00 \pm 0{\cdot}18$.

6. The second estimate of stress

The stress estimation in §4 involved a heat flux derived from the covariance between the temperature and the vertical velocity component. It is obviously desirable particularly for shipboard use to eliminate this type of measurement altogether and to use heat fluxes derived as in §5. If we write

$$a = D_u(r)\, r^{-\frac{2}{3}}, \quad b = D_T(r)\, r^{-\frac{2}{3}},$$

then, using the proposed value of 1·12 for the constant in (7), it follows that

$$\epsilon = 0{\cdot}5a^{\frac{3}{2}}, \quad H = -0{\cdot}893\frac{c_p \rho b \epsilon^{\frac{1}{3}}}{\partial\bar{\theta}/\partial z},$$

and hence, by the use of (4), we derive a new estimate, T_2, for the shearing stress. This has been done for those 27 observations for which both temperature and velocity information was available, and it was found that the mean value of T_2/τ was $0{\cdot}99 \pm 0{\cdot}15$. The individual values of T_2/τ are given in table 3.

Date	Run no.	T_2/τ	Date	Run no.	T_2/τ	Date	Run no.	T_2/τ
16. i. 52	6	0·44	25. ii. 52	15	1·07	3. iii. 52	6	0·25
21. i. 52	4	1·34	25. ii. 52	17	0·60	3. iii. 52	10	1·12
4. ii. 52	2	1·06	25. ii. 52	19	0·70	3. iii. 52	16	1·15
4. ii. 52	4	2·05	26. ii. 52	37	1·98	4. iii. 52	48	2·45
4. ii. 52	6	3·40	26. ii. 52	39	0·63	19. iii. 52	10	1·29
13. ii. 52	9	0·16	26. ii. 52	46	0·73	19. iii. 52	12	0·67
25. ii. 52	6	0·29	26. ii. 52	48	0·68	19. iii. 52	16	0·49
25. ii. 52	11	0·67	26. ii. 52	50	0·96	9. vii. 52	1	0·22
25. ii. 52	13	0·08	3. iii. 52	4	1·86	9. vii. 52	3	0·35

TABLE 3. Values of T_2/τ.

During a 5 min observation period it is not likely that all the assumptions made in deriving the expression for T_2 will hold good. Those of horizontal uniformity and steadiness in time, in particular, may break down over short periods. However, the coefficient of correlation between T_2 and τ is 0·49 which is significant at the 1 % level, while the coefficient of regression of T_2 on τ is $1{\cdot}00 \pm 0{\cdot}36$. Figure 1 shows the T_2 and τ values with the calculated regression line.

In the practical use of the structure-function method it is quite reasonable that a number of 5 min runs—or of runs over a longer period of observation—should be taken in order to average out the effects referred to above. The values of T_2 and τ were therefore subdivided into groups according to the magnitude of τ, and mean T_2 and τ were calculated for each group as shown in table 4. The coefficient of correlation between the two groups of means is 0·99.

A further subdivision of the T_2 results according to Richardson number at 1·5 m ($Ri_{1{\cdot}5}$) was made and is shown in table 5.

Application of the method of combining probabilities due to Fisher (1948, p. 99) indicates that the probability of the set of deviations of T_2/τ from unity being due to chance is 0·03 and it appears, therefore, that the trend of T_2/τ with stability shown in table 5 is significant.

This trend may, in part, be explained in terms of instrumental response characteristics. T_2 and τ were obtained ultimately from the same galvanometer records but, as stability increases, the turbulent energy moves further and further into smaller eddy sizes. Since the eddies contributing to the structure function measurement are smaller than the strongly anisotropic ones entering into the

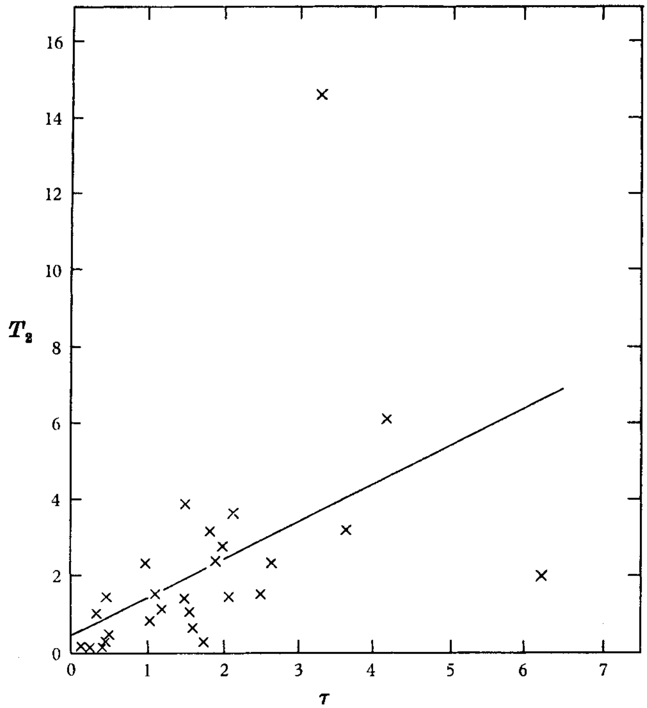

FIGURE 1. Comparison of stresses measured by two methods (individual 5 min runs).

Group	No. of observations	Mean τ	Mean T_2
A	5	0·29	0·30 ± 0·15
B	6	0·86	1·24 ± 0·26
C	5	1·56	1·41 ± 0·65
D	6	2·06	2·49 ± 0·34
E	5	3·98	5·63 ± 2·36

TABLE 4. Means of T_2 and τ, according to magnitude of τ.

Group	No. of observations	Mean $Ri_{1.5}$	Mean T_2/τ
F	6	+0·029	0·53 ± 0·15
G	5	−0·007	0·59 ± 0·15
H	6	−0·015	0·84 ± 0·24
J	5	−0·031	1·38 ± 0·29
K	5	−0·087	1·72 ± 0·52

TABLE 5. Values of T_2/τ, according to stability.

covariances, it might well be expected that the failure of the galvanometers to respond fully to the fine detail of the turbulence would affect T_2 more strongly than τ and the more so as stability increases. However, it would also be expected that this effect would cause T_2 to be less than τ in all conditions of stability so that all the features of table 5 cannot be explained in this way. This fact, together with the rather large difference between the means of T_1/τ and T_2/τ, make it unlikely that the values of the constants used in equations (3) and (7) can yet be regarded as firm for use in the lower atmosphere.

7. Conclusion

The results of the previous sections are encouraging and suggest that it may be possible to estimate stress or heat flux from a group of some thirty 5 min measurements of structure function with a standard error of 15–20 %. More observations, specially designed for the purpose, need to be made, however, to get more secure values for the two constants of proportionality and to elucidate stability effects, particularly in so far as they concern the instrumental response characteristics required.

It is worthy of note that equation (12) is essentially a conservation equation and that a similar equation could be constructed for water-vapour concentration. A structure-function method, involving mean-square humidity differences, could thus probably be developed for the assessment of the vertical turbulent flux of water-vapour, that is, the rate of evaporation from the underlying surface.

I have had the benefit of much discussion with Dr U. Radok and Mr E. K. Webb. Mr E. L. Deacon provided information on the probable errors in the reference stresses and the differential-analyser calculations of the autocorrelations were carried out under the direction of Mr N. E. Bacon.

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